Exam 5: Chapter 9
Math 176, Precalculus, Section 6265
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NAME Answer Key
NO TI-89 CALCULATORS!!!!!!!!

<u>100 points. Show all work and steps taken to arrive at the solution to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!</u>
Read and follow all directions careful. If a particular method is indicated, you must correctly solve the problem using THAT method to earn full points. Although many of the problems require that you solve them BY HAND, you may use your graphing calculator to verify your solutions.

Given the following matrices, carry out the indicated algebraic operations BY HAND or EXPLAIN why it cannot be performed. SHOW ALL WORK. (*30 points*)

$A=\begin{bmatrix}2 & -5\\ 0 & 1\end{bmatrix}$ $B=\begin{bmatrix}4 & 8\\ 1 & 2\end{bmatrix}$ $C=\begin{bmatrix}0 & -2 & 7\\ 5 & -1 & 3\end{bmatrix}$ $D=\begin{bmatrix}8 & -\frac{1}{2} & 0\\ 4 & 10 & -1\end{bmatrix}$ $E=\begin{bmatrix}4 & 1 & 0\\ -2 & -1 & 1\\ 4 & 0 & 3\end{bmatrix}$

1. $2(A-B)$

$2\begin{bmatrix}-2 & -13\\ -1 & -1\end{bmatrix}$

$= \boxed{\begin{bmatrix}-4 & -26\\ -2 & -2\end{bmatrix}}$

2. BC $= \begin{bmatrix}40 & -16 & 52\\ 10 & -4 & 13\end{bmatrix}$

3. $\det(B)$ $=0$

$4(2)-(8)(1)=0$

4. B^{-1} DOESN'T EXIST BECAUSE B IS SINGULAR

$\det(B)=0$

5. D^2 DOESN'T EXIST

$(2\times3)(2\times3)$

– CAN'T MULTIPLY

6. A^{-1} $= \frac{1}{\det A}\begin{bmatrix}1 & 5\\ 0 & 2\end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix}1 & 5\\ 0 & 2\end{bmatrix}$

$A^{-1} = \begin{bmatrix}\frac{1}{2} & \frac{5}{2}\\ 0 & 1\end{bmatrix}$

7. (8 pts) det(E) (Use the matrix E defined on the previous page and expand the determinant by the row of your choice)

$$\begin{bmatrix} 4 & 1 & 0 \\ -2 & -1 & 1 \\ 4 & 0 & 3 \end{bmatrix} = 4\begin{vmatrix} -1 & 1 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} -2 & 1 \\ 4 & 3 \end{vmatrix}$$

$$= 4(-3) - 1(-6+4)$$

$$= -12 + 10 = \boxed{-2}$$

8. (10 pts) Given the system of equations $\begin{cases} 5x + 8y - z = 64 \\ x + 4y + 7z = 56 \\ -9x + y + z = -73 \end{cases}$

a. Convert to a matrix equation.

2

$$\begin{bmatrix} 5 & 8 & -1 \\ 1 & 4 & 7 \\ -9 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 64 \\ 56 \\ -73 \end{bmatrix}$$

b. Solve using Cramer's Rule. Set up the solution by hand, then you may use your calculator to compute the required determinants. (Show the determinants.)

$$D = \begin{vmatrix} 5 & 8 & -1 \\ 1 & 4 & 7 \\ -9 & 1 & 1 \end{vmatrix} = -564$$

1 EACH

$$x = \frac{|D_x|}{|D|} = \frac{-5076}{-564} = 9$$

$$D_x = \begin{vmatrix} 64 & 8 & -1 \\ 56 & 4 & 7 \\ -73 & 1 & 1 \end{vmatrix} = -5076$$

$$y = \frac{|D_y|}{|D|} = \frac{-1692}{-564} = 3$$

$$D_y = \begin{vmatrix} 5 & 64 & -1 \\ 1 & 56 & 7 \\ -9 & -73 & 1 \end{vmatrix} = -1692$$

$$z = \frac{|D_z|}{|D|} = \frac{-2820}{-564} = 5$$

$$D_z = \begin{vmatrix} 5 & 8 & 64 \\ 1 & 4 & 56 \\ -9 & 1 & -73 \end{vmatrix} = -2820$$

$$\boxed{(9, 3, 5)}$$

9. (10 pts) Solve the system of nonlinear equations using the method of your choice.

$$\begin{cases} x^2 + y^2 = 4 \\ y = x^2 - 4 \end{cases}$$

$x^2 + (x^2 - 4)^2 = 4$

$x^2 + x^4 - 8x^2 + 16 = 4$

$-4 \quad -4$

$x^4 - 7x^2 + 12 = 0$

$(x^2 - 4)(x^2 - 3) = 0$

$(x-2)(x+2)(x+\sqrt{3})(x-\sqrt{3}) = 0$

$(2, 0)$

$(-2, 0)$

$(\sqrt{3}, -1)$

$(-\sqrt{3}, -1)$

10. *(2 pts each)* TRUE or FALSE. Determine whether each statement is True or False

a. An independent system of equations has infinitely many solutions. FALSE

b. Every square $n \times n$ matrix has an inverse matrix FALSE

c. Matrix multiplication is commutative, that is $AB = BA$. FALSE

d. Matrix addition is commutative, that is $A + B = B + A$. TRUE

e. If $\det(A) = 0$, then A^{-1} exists. FALSE

11. (10 pts) Given the system of equations $\begin{cases} x+y+z=0 \\ 3y+6z=3 \\ -7y-15z=-3 \end{cases}$.

a. Write the 3×4 augmented matrix associated with this system.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 3 & 6 & 3 \\ 0 & -7 & -15 & -3 \end{array}\right]$$

2

b. BY HAND, solve the system of linear equations using Gaussian elimination (Reduce to row-echelon form and solve). SHOW WORK!!!

$$R_2/3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & -7 & -15 & -3 \end{array}\right] \quad 7R_2+R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & 4 \end{array}\right]$$

8

$z=-4$

$y+2(-4)=1$

$y=9$

$x+9-4=0$

$x+5=0$

$x=-5$

$\boxed{(-5, 9, -4)}$

12. (10 pts) Write the partial fraction decomposition of $\frac{7x^2 - x + 15}{x^3 + 5x}$.

$$\frac{7x^2 - x + 15}{x(x^2+5)} = \frac{A}{x} + \frac{Bx + C}{x^2+5}$$

$$7x^2 - x + 15 = A(x^2+5) + x(Bx + C)$$

$$= Ax^2 + 5A + Bx^2 + Cx$$

$\underline{x^2}$: $7 = A + B$ $\qquad$ $A = 3$

$\underline{x}$: $-1 = C$ $\qquad$ $7 = 3 + B$

$\underline{1's}$: $15 = 5A$ $\qquad$ $B = 4$

$$= \boxed{\frac{3}{x} + \frac{4x - 1}{x^2+5}}$$

13. (12 pts) A company makes three kinds of cable. Cable A requires 3 black, 3 white, and 2 red wires. Cable B requires 1 black, 2 white, and 1 red. Cable C requires 2 black, 1 white, and 2 red wires. If 100 black, 110 white, and 80 red wires were used, how many of each type were used? Use the method of your choice. All matrix operations should be done with the calculator, but you must clearly state the steps you took to arrive at your solution.

$\underline{\text{BLACK}}$: $3A + 1B + 2C = 100$

$\underline{\text{WHITE}}$: $3A + 2B + 1C = 110$

$\underline{\text{RED}}$: $2A + B + 2C = 80$

$$\underbrace{\begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} A \\ B \\ C \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 100 \\ 110 \\ 80 \end{bmatrix}}_{B}$$

$A^{-1}B = X$

$\boxed{\text{CABLE A: 20} \quad \text{CABLE B: 20} \quad \text{CABLE C: 10}}$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ -4/3 & 2/3 & 1 \\ -1/3 & -1/3 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 110 \\ 80 \end{bmatrix} = X = \begin{bmatrix} 20 \\ 20 \\ 10 \end{bmatrix}$$

BONUS (total of 10 extra points)

(5 pts) With the aid of your CALCULATOR, solve the system of equations using one of the following methods: Cramer's Rule, Gauss-Jordan elimination, or Matrix Inverse. All matrix operations should be done with the calculator, but you must clearly state the steps you took to arrive at your solution.

$$\begin{cases} x+\ y+\ z-\ w=6 \\ 2x-\ y+3z+4w=-4 \\ 4x+2y-\ z-\ w=-13 \\ -x-2y+4z+3w=12 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 2 & -1 & 3 & 4 \\ 4 & 2 & -1 & -1 \\ -1 & -2 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ -13 \\ 12 \end{bmatrix}$$

$$AX = B \qquad X = A^{-1}B$$

$$A^{-1} = \begin{bmatrix} -0.785714 & -0.642857 & 1 & 0.928571 \\ 2.07143 & 1.78571 & -2 & -2.35714 \\ 0.357143 & -0.071429 & 0 & 0.214286 \\ 0.642857 & 1.07143 & -1 & -1.21429 \end{bmatrix}\begin{bmatrix} 6 \\ -4 \\ -13 \\ 12 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 5 \\ -2 \end{bmatrix}$$

$$\boxed{(-4, 3, 5, -2)}$$

(5 pts) Find the inverse of the following matrix BY HAND!!! $\begin{bmatrix} 1 & 3 & -1 \\ 1 & 4 & 0 \\ -1 & -3 & 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -1 & 1 & 0 & 0 \\ 1 & 4 & 0 & 0 & 1 & 0 \\ -1 & -3 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2-R_1 \\ R_1+R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 3 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_1+R_3 \\ R_2-R_3 \\ R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & -2 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \quad R_1-3R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & -3 & 4 \\ 0 & 1 & 0 & -2 & 1 & -1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\boxed{\begin{bmatrix} 8 & -3 & 4 \\ -2 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}}$$